COMBINATORIAL STUDY OF NUCLEAR LEVEL DENSITY FOR ASTROPHYSICAL APPLICATIONS

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Abstract : We present some results and remarks based on a combinatorial approach of the evaluation of the nuclear level density. First, we show that it is possible to extract some reliable information from the output of the program whose rough data present a strong statistical fluctuation from bin to bin. This includes smoothing and evaluation of the desired quantities. After some comments about the spin and parity distributions, we consider the question of the non-equipartition of parities, mainly, at low energies. Finally, we present a simple model to include and test this effect in the computation of thermonuclear reaction rates.

1 : INTRODUCTION

Nuclear level densities are one of the suitable quantities for nuclear physics and, especially, for nuclear reaction models (e.g. the statistical Hauser-Feshbach theory). Therefore, they have been investigated a long time ago, the first attempt, using the infinite non-interacting Fermi-gas model, being that of Bethe in 1936 [1].

The need to evaluate reliable reactions rates, and thus nuclear level density, for numerous nuclei arises from the specificity of nuclear astrophysics which deals with nuclei in the whole chart of nuclides, towards the proton drip line (for α -p or r-p process), near both sides of the valley of stability (for p and s processes) or towards the neutron drip line (for the r - process). In all these cases, very few reaction rates are measurable experimentaly, especially at the energies of astrophysical interest. For all the others, only theoretical predictions are possible.

For this purpose, we must develop some *ab initio*, but rather simple, methods (in order to minimize the computational work) with the minimum number of parameters. This is particularly the case of the nuclear level density.

The aim of this paper is to present a combinatorial method of computing the level density by enumeration of all the possible configurations, and to obtain easily a simple law describing the energy dependence of the level density. Beyond the spin distribution, we focus our attention onto the parity distribution which, partially, originates this work. The final intent is to evaluate the various uncertainties, constraints or difficulties related with the level density problem, and their impact on reaction rates predictions.

Some of the previous attempts are briefly reviewed and commented in §1. Our attempt is depicted in §2 with the description of the model and the preliminary treatments. Some results, including the determination of the level density parameter and a comparison with the usual spin distribution function, are shown in §3. The problem of equipartition of the parity distribution for the lowest energy levels is presented in §4. In our conclusions (§5), we consider some possible developments or applications, a few of them (mostly astrophysical ones) being now in progress. The technical part of our configuration generator algorithm will be described elsewhere.

There are basically two main classes of methods for evaluating the nuclear level densities. The first methods, and the most used, are based on the statistical approach via the partition function (see e.g. refs. [2], [3] for review, or [4], [5] for computational programs). The other method is the combinatorial approach which has been much less investigated, mainly due to the extent of the computations it requires. Now, let us focus on this last one.

To our knowledge the first attempt is due to Hillman and Grover [6]. They used different sets of single particle levels, the pairing energies being evaluated within the BCS approximation. In the same spirit, a recursive method has been deviced

by Williams [7], that represents an interesting alternative way of computing level densities (see §2-c).

Another attempt was done by Ford [8]. He used Woods-Saxon or Nilsson single particle levels with a new combinatorial algorithm to generate the configurations. This algorithm works according the $M-$scheme. This implies special and supplementary assumptions in order to obtain the $J-$scheme configuration, and thus, the spin distribution and, more importantly, the energy of the configuration under consideration (see §2).

2 : THE PRESENT ATTEMPT

a — Description of the model

As previously said, the combinatorial method is based on a set of single particle levels, on an algorithm to generate configurations, and on an evaluation of various physical quantities of each configuration under study such as energy, parity and spin distribution (due to degeneracy).

The results presented here have been obtained with the use of spherical single particle levels derived within the framework of an energy density approximation described in ref. [9]. We do not investigate here the influence on the level density of a change of those single particle spectra. This question will be examined elsewhere along with the consideration of deformed single particle spectra.

In order to generate all the configurations up to a given maximum energy (chosen less or equal to 30 MeV), we have adapted Ford's algorithm to our specific purposes. In fact, the algorithm designed by Ford explores all the possible configurations by increasing energies. Hence, it avoids the evaluation of unnecessary physical quantities for configurations with energy far in excess of the adopted upper limit.

The $M-$scheme used in Ford's algorithm [8] to classify the configurations has been converted into the desired $J-$scheme. This transformation is complicated by the fact that many "$M-$configurations" lead to the same "$J-$configuration" (see the example given in table 1). We have also added to the original algorithm in order to keep only the first occurrence of a new "$J-$configuration" ; the following "$M-$configurations" that lead to a previous studied "$J-$configuration"being skipped. The technical details of our modified algorithm will be presented elsewhere.

The pairing is treated following the idea originated by Ford [8]. Namely, each configuration is inspected subshell by subshell. In each one, we examine all the possibilities to form pairs with the available nucleons. Each time that a nucleon pair can be formed, we subtract the so-called "pairing energy" E_p (as defined by Ford). Such a procedure looks like a seniority scheme applied subshell by subshell to the whole configuration.

Unfortunately, when considering a non-vanishing pairing energy E_p , his algorithm fails. More precisely, it produces an erroneous spin distribution as can be seen easily from a simple example. Let us consider the $(1d_{5/2})^3$ configuration with a pure δ pairing

force $E_p \times \delta_{J=0}$. With the help of the Mayer-Jensen'table (see e.g. ref. [6]), we find a "ground" state $J = \frac{5}{2}$ and two "degenerate" states $J = \frac{3}{2}$ and $J = \frac{9}{2}$. If we follow Ford's procedure, we obtain all the $(m_1 m_2 m_3 | M)$ states (see table 1). In order to find the ground state, we collect, as suggested, the $M-$states by pairs (one m_i and its opposite). Thus, we find the following states and their corresponding spin (see table 1, last column). Considering that the number of states with spin J is equal to the number of states with $M = J$ minus the number of states with $M = J + 1$, we obtain a twofold degenerate "ground" state with $J = \frac{5}{2}$ and an "excited" state with $J = \frac{9}{2}$ (for which the only projections $M = \pm \frac{9}{2}$ and $M = \pm \frac{7}{2}$ exist ..., the other values of the magnetic quantum number being counted as spurious in one of the $J = \frac{5}{2}$ "ground" state).

We have considered a number of parametrizations for the "pairing" energy E_p (expressed in MeV) :

- $E_p = 0$ *i.e.* no pairing   (1a)
- $E_p = 9.6 A^{-1/2}$ as used by Ford   (1b)

                                                                      ref. [8]

- $E_p = \begin{cases} 7.36 A^{-1/3}(1 - 8.15\eta^2) & \text{for neutrons} \\ 7.55 A^{-1/3}(1 - 6.07\eta^2) & \text{for protons} \end{cases}$ (1c)

                                                           ref. [10]

where $\eta = \frac{N-Z}{A}$ is the neutron – proton asymmetry (see also refs. [11], [12], [13] and the well known droplet pairing energy).

In all cases, E_p is identified with the pairing term of a nuclear mass formula, generally assumed to be equal, with the appropriate signs, to the back-shift δ (see eq.(4)). In addition, we assume that E_p does not vary with the excitation energy. The simplest way to avoid this approximation is to compute an effective pairing energy E_p for *every* single particle subshell. To do this, we may consider a (fictitious) nucleus with all subshells filled up to the subshell under consideration. The pairing energy of *this* subshell can be evaluated from the energy difference between the configuration energies computed with and without pairing interaction. Such an improvement is able to take into account the fact that the pairing energy vanishes at high energy.

For a given "$J-$configuration", a Mayer-Jensen table (see ref. [6]) provides the spin distribution for each group of states, sorted by energy as done by the treatment of the pairing energy. Of course, in the $E_p = 0$ case, there is only one set of states at the same energy, and hence the degeneracy of this level is maximum even for the "ground" state.

Once the proton and neutron levels are obtained, they have to be convoluted in order to obtain the final distribution of nuclear levels. For the coherence of our assumptions, no residual $p - n$ interaction has been considered.

b — Preliminary treatments

For each energy bin, from 0 MeV up to 30 MeV by 10 keV steps, the algorithm described in the previous section provides the number of states of given spin (from $J = 0$ or $J = \frac{1}{2}$ to $J = 60$ or $J = \frac{121}{2}$) with the indication of parity. As we see on figure 1a, there is, as expected, a strong statistical fluctuation from bin to bin. It is necessary to treat these data in order to extract a reliable information, like the level density parameter a (see §3-c).

Let us notice that the aim is to smooth data in order to be able to extract parameters for some fits (see §3-a), and then, using them to compute the nuclear level density at any energy. The goal is **not** to smooth data in order to evaluate, by some interpolation, the density at an arbitrary energy from the smoothed data. In fact, the observed ripples prevent such a treatment : for example, the resulting density might not be a monotonically increasing function of the energy ! Thus, several smoothing methods have been tested [14].

$-1-$ "The minimum - maximum - median method" (hereafter 3m-method). Each bin is associated with the set of its neighbors (typically a few tens on both sides). Each set is sorted in increasing order and, some extreme points being possibly discarded (for example to take account of empty bins). In the remaining points, either the first point (the minimum), or the last point (the maximum) or the middle point (the median) is selected. This point represents the value of the smoothed data for the bin under consideration.

One of the advantages of this method is its high controlability on the data treatment. Unfortunately, it requires a lot of computational time, due to the necessary sorting. Thus, we have turned our attention to faster methods. For instance :

$-2-$ The convolution of the input distribution with the Fejer's function (*viz.* $\propto$ $\left(1 - (x/n)^2\right)^2$) which guarantees the smoothing to be compatible with the evaluation of the first derivative. This can be useful when dealing with the cumulative number of levels, and not directly with the level density.

$-3-$ The running mean method based on the convolution of the input distribution with a rectangular step function. This simple method has been adopted here (the average being taken over 100 keV). The result of our smoothing is presented on figure 1b for the same nucleus as in figure 1a. Notice that the cumulative number of levels, involving a quadrature, provides a smoother curve than the level density itself, but the extraction of parameters would be less reliable and easy.

c — Other possible models

As noted previously by various authors (see, for instance, Hillman and Grover [6] or Ford [8]) the computational time represents a quite severe restriction to the use of the combinatorial method. The recursive method proposed by Williams [7] is, in this respect, very efficient but is not directly applicable to handle the parity and the spin distributions.

Recently, a Monte-Carlo approach has been developped [15]. Using such a method, one may obtain similar results with some advantages, for instance, a short computational time which could allow the use of more sophisticated treatments of pairing. In addition, it allows also to take into consideration residual interactions between neutrons and protons. However, there are some disadvantages, such as the need to test the sampling function, to calibrate results with other methods, like the full combinatorial method described here. We think that the Monte-Carlo method will be very suitable for the heaviest nuclei, where the combinatorial method is still computationally unworkable, whereas our full combinatorial method represents the best compromise for moderate heavy nuclei, when the desired number of levels does not exceed a few millions and when the Monte-Carlo approach may suffer from lack of statistics.

3 : RESULTS AND REMARKS ABOUT LEVEL DENSITY

We have used our model for four sets of nuclei :
— ^{40}Ca , ^{44}Ti , ^{46}Ti , ^{48}Cr and ^{50}Cr , called the "magical" series since their study allows the evaluation of various physical parameters and their evolution with the distance from a doubly magic nucleus,
— an odd A isobaric series : ^{47}K , ^{47}Ca , ^{47}Sc , ^{47}Ti and ^{47}V ,
— an even A isobaric series : ^{50}K , ^{50}Ca , ^{50}Sc , ^{50}Ti , ^{50}V , ^{50}Cr , ^{50}Mn and ^{50}Fe ,
— for comparison purposes, the nuclei already studied by Ford, *i.e.* ^{56}Fe , ^{59}Co , ^{60}Ni , ^{62}Ni , ^{61}Cu and ^{63}Cu .
The two isobaric series have been adopted for the purpose of comparison with the results of Pieper [16].

For all these nuclei, the distributions of spins and parities have been evaluated up to 30 MeV. As expected, the computational time varies much from a nucleus to another : the computer time needed varies typically (for a VAX8600) from 40 mn up to 1 day. This indeed represents a very drastic limitation of the use of such a method.

a — Comparison with nuclear level density formulae

After smoothing the results derived from the algorithm described in §2-b, we can try to extract some parameters that appear in classical level density formulae. Here, we consider the so-called "Bethe back-shifted Fermi gas formula" (see, for instance, ref. [8] appendix B) :

$$\rho(E) = \frac{1}{\sqrt{2\pi}\sigma} \frac{\sqrt{\pi}}{12a^{1/4}} \frac{\exp(2\sqrt{aU})}{U^{5/4}} \tag{2}$$

where $\rho(U)$ is the level density at energy $U = E - \delta$, δ being the back-shift, introduced to take into account the differences between nuclei according to evenness or oddness of N and Z, a is the so-called "level density parameter", and σ is related to the spin-distribution.

If the classical energy dependence of σ (see §3-e, eq.(11)) is adopted, eq.(2) transforms into :

$$\rho(E) \propto U^{-3/2} \, \exp\left(\sqrt{2aU}\right) \tag{3}$$

We have investigated the change of the exponent $3/2$ of U in eq.(3) by $3/2 + x$, where x is constant. It appears that only $|x| < 0.1$ is compatible with data, so that the conventional eq.(3) has been retained.

b — Comparison between some determinations of the back-shift δ

First, the determination of δ proves to be very difficult from our data and with our treatment. In fact, the cumulative number of states $N(E)$ (that minimizes the statistical fluctuations, see figure 1b) cannot provide any information on this parameter, since, whatever δ , $N(E = 0) = 0$. One has thus to work with the level density itself. Then, the main drawback is that the large statistical fluctuations prevents a reliable

determination of δ . In such conditions δ is assumed to be equal to the pairing energy of the liquid drop nuclear mass formula, so that :

$$\delta = \frac{1}{2} \left[(-1)^N E_p(n) + (-1)^Z E_p(p) \right] \qquad (4)$$

where $E_p(n)$ and $E_p(p)$ are the neutron and proton pairing energies. This assumption is coherent with our treatment of the pairing. The results we present in this paper are determined with the pairing energies given in eq.(1c).

For comparison, we consider the phenomenological determination [17] :

$$\delta = \Delta - 5.6A^{-0.28} \qquad (5)$$

where

$$\Delta = \frac{1 + (-1)^N}{2} \Delta_n + \frac{1 + (-1)^Z}{2} \Delta_p \qquad (6)$$

Δ_n and Δ_p being the experimental neutron and proton pairing energies (see ref. [18]).

Table 2 compares the values of δ extracted from eq.(5) with those determined in this work (see eq.(4)). The observed discrepancies do not exceed 0.5 MeV from the average value, and can be considered as quite negligible with respect to the uncertainties arising from the determination of the level density parameter a (see §3-d).

c — The level density parameter a

For the nuclei considered here, Mathias [14] has determined the best values of a that reproduce our combinatorial data. These values are given in table 3, and agree to better than 10 % with various experimental determinations.

d — Accuracy of the level density

Let us analyse the origins of possible inaccuracies in the determination of the level density. Considering only the exponential term, we can write :

$$\frac{\Delta\rho}{\rho} = \sqrt{aU} \left(\frac{\Delta a}{a} + \frac{\Delta\delta}{U} \right) \qquad (7)$$

The back-shift δ gives an arror that is large at low energies, but that vanishes for higher energies. For instance, with the maximum error quoted previously (*i.e.* 500 keV), the error arising from the determination of δ does not exceed 40 % for an excitation energy of 10 MeV if $a \simeq 6$ MeV^{-1}, which is a typical value for the nuclei considered here.

The relative error arising from the determination of a remains constant. Table 3 shows that $\frac{\Delta a}{a} \simeq 0.15$ leads to a factor of 2 uncertainty in the level density at 10 MeV.

Hence, as long as a remains the most difficult quantity to predict for exotic nuclei, the level density will suffer an uncertainty factor of, at least, two.

e — The spin distribution and the spin cut-off parameter σ

The level density, as a function of energy and spin can be written as :

$$\rho(E, J) = f(U, J)\, \rho(E) \tag{8}$$

where $\rho(E)$ is the level density given by eq.(2), and $f(U, J)$ is the spin distribution at energy E. It is expressed as :

$$f(U, J) = \frac{2J+1}{2\sigma^2} \exp\left(-\frac{J(J+1)}{2\sigma^2}\right) \tag{9}$$

where σ is the "spin cut-off".

We have checked this well known gaussian distribution. As expected, it fits very well the calculated spin distribution at any energy, provided, of course, that a sufficient number of states exists at the considered energy (see figure 2).

The calculation of the corresponding value of σ can be based on the fact that eq.(9) presents a maximum at $J \simeq \sigma - \frac{1}{2}$. Another method consists in the computation of the average of $J(J+1)$ over the spin distribution. In the limit of a infinitely large number of spins, it leads to :

$$\sum_{J=0}^{J=J_{max}} J(J+1)\, f(U, J) \simeq \int_0^\infty J(J+1) f(U, J) dJ = 2\sigma^2 \tag{10}$$

Figure 3 presents values of σ for a large range of energy bins. For our 10 keV bin width value, we notice a large dispersion of points which prevents any determination of a constant value of σ (as guessed by ref. [17]), or any check of the "classical solid rotator law" :

$$\sigma^2 = \frac{\theta_{rigid}}{\hbar^2} \sqrt{\frac{U}{a}} \tag{11}$$

θ_{rigid} being the moment of inertia.

Let us note that an increase of the width of the energy bins reduces the dispersion of points, the new values of σ being slightly greater than the previous ones. In fact, each of these new values σ_{new} verifies the relation $\sigma_{new}^2 = \sum_i \sigma_i^2$, where the summation runs over the old energy bins.

Our determinations of σ can be compared with the constant value :

$$\sigma = (0.98 \pm 0.23)\, A^{(0.29 \pm 0.06)} \tag{12}$$

proposed in ref. [17] from the data of the first excited levels. For instance, we obtain $\sigma \simeq 5.$ at 13.5 MeV for for ^{50}Mn whereas eq.(12) leads to $1.84 \le \sigma \le 4.76$.

Taking into account the fact that –1– σ shows an overall increase with energy, as perceptible on figure 3, –2– eq.(12) is derived from the first excited levels and, –3– our chosen energy of 13.5 MeV is far from the energies of the first levels (but it ensures a very good gaussian distribution of spins) and we conclude that, a constant value of σ cannot be rejected at least for energies below about 20 MeV.

Let us notice that, whatever the width of the bins can be (say less than 500 keV, typically 100 keV, in order to have enough energy points), there are some bins with σ values less than the neighboring ones. Hence, the series of $\{\sigma_i\}$ is not monotonic, and *all the narrower* the bins are. However, as already stated, the overall trend is the increase of the upper and lower limits of the σ values with energy (see figure 3). We do not attempt here to evaluate quantitatively the variations of these upper and lower bounds with energy or bin width (these two functions being increasing functions in both variables). Instead, we limit ourselves to the check of the $U^{1/4}$ dependence predicted in eq.(11).

To do that, we have analysed the values σ for the energy bins for which a **new** maximum value in the spin distribution ("yrast spin") appears for the **first** time. The interest of such a procedure is that the resulting values of σ and energies (represented by big black triangles on figure 3) are independent of the bin width.

Apart from one point, σ appears to increase with energy, and a power law fit gives an exponent lying between 0.12 and 0.17 . A Monte-Carlo calculation with bins of 1 MeV up to an excitation energy of 100 MeV (see ref. [19]) is in good agreement, with the $U^{1/4}$ law, as expected in the high energy limit. We conclude that the discrepancy found here originates from the considered lower energies which are necessary for astrophysical purposes.

4 : THE PARITY DISTRIBUTION

a — The problem of the equipartition of the parities

Following Ericson [2], it is classicaly assumed that the equipartition of the parities holds at "sufficiently" high energy so that we can write :

$$\rho(E, J, \pi) = \frac{1}{2} \ f(U, J) \ \rho(U) \tag{13}$$

Further work has been devoted to that question (see, for instance, ref. [20]), but has been poorly investigated in the framework of combinatorial level density calculations.

The question of the parity distribution is important in a variety of problems, and especially in the evaluation of the transmission coefficients in the γ channel that appears in the Hauser-Feschbach theory.

b — Results

The parity distribution at a given energy can be described in terms of the asymmetry ratio :

$$A(E) = \frac{N^+(E) - N^-(E)}{N^+(E) + N^-(E)} \tag{14}$$

where N^π are the number of levels with parity $\pi = \pm 1$. Of course $A(E = 0) = \pi_g$ where π_g is the parity of the ground state, and the equipartition of parities is realized when $A(E) \simeq 0$.

Figures 4a and 4b show the evolution of $A(E)$ with energy for the even-even nucleus ^{56}Fe and for the odd-odd nucleus ^{50}Mn . Obviously, $A(E)$ tends to 0 more rapidly in the latter case. Clearly, these figures show that the hypothesis of parity equipartition is invalid below energies less than about 15 MeV, in contrast with previous predictions.

We now device a simple model in order to evaluate quantitatively the critical energy above which $A(E) \simeq 0$.

c — A simple model

In the framework of the shell model and considering only particle or particle-hole intrinsic excitations with no interaction mixing, we introduce different energies for protons (p) and neutrons (n) :

$-1-$ $D_{(n)}$ and $D_{(p)}$ are the energy differences of the last filled single particle level *splev*, called for simplicity the Fermi *splev*, and the first *splev* with a parity opposite to the Fermi *splev* , noted $\pi^0_{(n)}$ and $\pi^0_{(p)}$. $D_{(n)}$ and $D_{(p)}$ are counted positive for a single particle excitation, or negative for a particle-hole excitation.

We consider that, from 0 MeV to $E_1 = \min(|D_{(n)}|, |D_{(p)}|)$, there is no way to form levels with a parity different from the ground state one, π_g . Hence, $A(E) = \pi_g$ over this interval. This assumption is strongly supported by the fact that, at low energy, the parity of individual states is, most of the time, experimentally known and remains constant for the first few states. Taking into account the parity of N and Z, we can improve somewhat the previous determination of E_1 by adding the corresponding pairing energies $E_p(n)$ or $E_p(p)$.

$-2-$ $D_{(2n)}$ and $D_{(2p)}$ are the energy differences between Fermi *splev* and the first *splev* with a parity equal to $\pi^0_{(n)}$ and $\pi^0_{(p)}$ that is not in the same major shell as the Fermi *splev*.

For excitation energies greater than $E_2 = \max(|D_{(2n)}|, |D_{(2p)}|)$, there are, at least, for protons and for neutrons, two sets of *splev* with opposite parities which are of comparable sizes. Therefore, we consider that $A(E) \simeq 0$ for $E \geq E_2$. Moreover, our simple model assumes that $A(E)$ goes to 0 linearly for $E_1 < E < E_2$ (see figure 5).

Table 4 presents the values of E_1 and E_2 determined from our simple model. This model seems able to reproduce the overall behavior of the parity distribution for most of the analyzed nuclei. A few exceptions exist however : ^{40}Ca , ^{44}Ti , ^{47}Ca and ^{50}Ca exhibit an additional ripple before $A(E)$ vanishes, but the physical interpretation of E_1 and E_2 remains the same.

From a practical point of view, since $A(E)$ tends to 0 when the energy increases, as expected, we can consider that the equipartition of parities is "sufficiently" achieved when the $|A(E)| < 0.25$. With this convention, the equipartition of parity is realized at an energy close to 15 MeV for ^{56}Fe , and between 12 MeV and 14 MeV for ^{50}Mn . Therefore, the equipartition of parities is not realized at energies of astrophysical interest at least, for light and moderately heavy nuclei.

5 : CONCLUSIONS

We have presented an *ab initio* tool to compute the nuclear level density in the framework of the combinatorial approach. We are able to predict the values of the level density parameter a and we give a prescription to evaluate the back-shift energy δ .

We underline the difficulty encountered in the evaluation of the spin cut-off parameter σ with energy. In fact, the choice of a constant value has not to be rejected and the usual $U^{1/4}$ law is not clearly established, at least at low energies. We suggest to determine upper and lower bounds of σ versus energy, according to the width of the bins.

One of the major questions raised in this work concerns the approach of the equipartition of the parity distribution with increasing energy. We have introduced two critical energies, physically based on the nuclear shell model, and we have presented a very simple model to sketch the evolution of the asymmetry ratio of the parity distribution with energy.

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Table 1

(m_1, m_2, m_3)						M	E	level
$-5/2$	$-3/2$	$-1/2$	$+1/2$	$+3/2$	$+5/2$			
*	*	*				$-9/2$	0	(1)
*	*		*			$-7/2$	0	(1)
*	*			*		$-5/2$	$-E_p$	(2')
*	*				*	$-3/2$	$-E_p$	(2')
*		*	*			$-5/2$	$-E_p$	(3')
*		*		*		$-3/2$	0	(1)
*		*			*	$-1/2$	$-E_p$	(2')
*			*	*		$-1/2$	0	(1)
*			*		*	$+1/2$	$-E_p$	(2')
*				*	*	$+3/2$	$-E_p$	(2')
	*	*	*			$-3/2$	$-E_p$	(3')
	*	*		*		$-1/2$	$-E_p$	(3')
	*	*			*	$+1/2$	0	(1)
	*		*	*		$+1/2$	$-E_p$	(3')
	*		*		*	$+3/2$	0	(1)
	*			*	*	$+5/2$	$-E_p$	(2')
		*	*	*		$+3/2$	$-E_p$	(3')
		*	*		*	$+5/2$	$-E_p$	(3')
		*		*	*	$+7/2$	0	(1)
			*	*	*	$+9/2$	0	(1)

Table 1 :   The $M-$configurations involved in the case of a $(1d_{5/2})^3$ configuration. The column M indicates the resulting value of the spin-projection. Here we find, as stated in Mayer-Jensen'table, that such a configuration leads to one state for each spin $J = 9/2$, $J = 5/2$ and $J = 3/2$.

The energy of each state, as evaluated by Ford's rule, is displayed in the next column where E_p is a given constant pairing energy. The last column indicates the identification of each state with a corresponding spin. Clearly, the states labelled (1) gives, unambiguously, a $J = 9/2$ spin with energy $E = 0$ (let us notice that this level is "incomplete" in the sense that the states $M = \pm 5/2$ are not present). The states labelled (2') and (3') give, with some unspecified linear combinations, the level (2) and (3) with the same spin $J = 5/2$ with energy $E = -E_p$.

Table 2

Nucleus	d_p	d_V
"Magic" series		
^{40}Ca	2.18	2.71
^{44}Ti	2.11	3.45
^{46}Ti	2.05	2.23
^{48}Cr	2.05	2.58
^{50}Cr	2.00	1.45
Isobaric $A = 47$ series		
^{47}K	−0.10	−0.58
^{47}Ca	0.07	−0.40
^{47}Sc	−0.05	−0.83
^{47}Ti	0.03	−0.44
^{47}V	−0.03	−1.36
Isobaric $A = 50$ series		
^{50}K	−1.20	−1.87
^{50}Ca	1.45	0.92
^{50}Sc	−1.66	−1.87
^{50}Ti	1.82	1.62
^{50}V	−1.93	−1.87
^{50}Cr	2.00	1.45
^{50}Mn	−2.02	−1.87
^{50}Fe	2.00	1.52
"Ford" series		
^{56}Fe	1.88	1.12
^{59}Co	−0.04	−0.58
^{60}Ni	1.84	1.44
^{62}Ni	1.76	1.72
^{61}Cu	−0.03	−0.65
^{63}Cu	−0.04	−0.53

Table 2 :   Values of the back-shift δ as determined in this work (*col.* d_p) and comparisons with other available values : from the phenomenological formulae of VonEgidy and collaborators [17] (*col.* d_V for their equation (11)) . For ^{50}K the values of the (experimental) pairing energies are not available and, therefore, the corresponding values for δ by the formulae of VonEgidy would not be calculable, but fortunately for an odd-odd nucleus such as this one, the term coming from the neutron and proton pairing vanishes. Let us note, that as usually assumed, the back-shift for odd A nuclei is compatible with 0 .

Table 3

Nucleus	a_0	a_2	a_u	a_d	a_D	a_V	a_p
"Magic" series							
^{40}Ca	6.87	6.15	3.7		3.6		
^{44}Ti	5.72	5.59					
^{46}Ti	5.35	5.31	5.0				
^{48}Cr	5.40	5.45					
^{50}Cr	5.22	5.09					
Isobaric $A = 47$ series							
^{47}K	5.42	5.28					
^{47}Ca	5.68	5.13					
^{47}Sc	4.93	4.96					
^{47}Ti	4.86	4.92	5.4	4.68	5.54		6.6
^{47}V	5.63	5.96	5.0				
Isobaric $A = 50$ series							
^{50}K	5.85	5.62					
^{50}Ca	5.97	5.49					
^{50}Sc	5.54	5.43					
^{50}Ti	5.46	5.07	5.5	5.53	5.26		
^{50}V	5.17	5.25					6.3
^{50}Cr	5.22	5.09					
^{50}Mn	5.35	5.65					
^{50}Fe	5.26	4.91					
"Ford" series							
^{56}Fe	6.83		5.8				
^{59}Co	6.30		6.0	5.50	6.31		
^{60}Ni	6.77		6.6				
^{62}Ni	6.65		6.7	6.48	7.27		
^{61}Cu	6.63			5.03	5.99		
^{63}Cu	6.50		7.0	5.74	6.63		

Table 3 : Values of the level density parameter a (in MeV^{-1}) as determined in this work (*col.* a_0 without pairing, *col.* a_2 with pairing, *cf.* eqs. (1c) and (4)) and comparisons with other available experimental values : from the publication [21] (*col.* a_u , from the unpublished Ph.D. thesis of J.A.Holmes), from the work of Dilg and collaborators [22] , the two values (*col.* a_d) and (*col.* a_D) presented have been computed according to the assumption made about the momentum of inertia (see the original reference for further details), from the work of VonEgidy and collaborators [16] (*col.* a_V) , from the work of Pieper [18] (*col.* a_p) .

Table 4

Nucleus	$D_{(n)}$	$D_{(p)}$	$D_{(2n)}$	$D_{(2p)}$	E_1	E_2
"Magic" series						
^{40}Ca	3.82	3.66	14.25	12.68	3.66	14.25
^{44}Ti	−4.01	−3.82	−15.73	10.88	3.82	15.73
^{46}Ti	−4.09	−3.87	−15.83	12.43	3.87	15.83
^{48}Cr	−4.17	−3.96	−15.83	11.68	3.96	15.83
^{50}Cr	−4.25	−4.01	−15.95	13.18	4.01	15.95
Isobaric $A = 47$ series						
^{47}K	−4.17	3.98	−16.16	14.25	3.98	16.16
^{47}Ca	−4.14	3.87	−16.06	14.00	3.87	16.06
^{47}Sc	−4.12	−3.87	−15.93	14.37	3.87	15.93
^{47}Ti	−4.12	−3.90	−15.91	13.27	3.90	15.91
^{47}V	−4.13	−3.91	−15.84	12.02	3.91	15.84
Isobaric $A = 50$ series						
^{50}K	6.51	3.90	15.51	14.04	3.90	15.51
^{50}Ca	6.52	3.86	15.85	13.88	3.86	15.85
^{50}Sc	6.66	−3.94	16.28	−15.06	3.94	16.28
^{50}Ti	−4.26	−3.99	−16.08	−15.24	3.99	16.08
^{50}V	−4.27	−4.01	−16.09	14.41	4.01	16.09
^{50}Cr	−4.25	−4.01	−15.95	13.18	4.01	15.95
^{50}Mn	−4.26	−4.03	−15.91	12.05	4.03	15.91
^{50}Fe	−4.26	−4.05	−15.81	10.94	4.05	15.81
"Ford" series						
^{56}Fe	5.85	−4.09	15.77	14.98	4.09	15.77
^{59}Co	5.20	−4.04	15.22	−15.01	4.04	15.22
^{60}Ni	5.10	−4.08	15.15	−15.06	4.08	15.15
^{62}Ni	2.97	−4.10	12.87	−14.88	2.97	14.88
^{61}Cu	5.13	5.80	15.16	11.47	5.13	15.16
^{63}Cu	2.90	5.61	12.78	12.49	2.90	12.78

Table 4 : Values of the parameters $D_{(n)}$, $D_{(p)}$, $D_{(2n)}$, $D_{(2p)}$, E_1 and E_2 used in our simple model (see text §4-c).

FIGURE CAPTIONS

Figure 1a : Rough data of density level (with summation over the spins and parities) for the nucleus of ^{56}Fe (with no-pairing energy).

Figure 1b :   The same data of figure 1a but after treatment by a running mean method over 51 points (*i.e.* 500 keV). The fitted curve with $a = 6.83$ MeV^{-1} is shown. Let us note that the level density is shown and not the cumulative number of levels.

Figure 2 : A typical spin distribution (solid curve) as calculated for the ^{50}Mn nucleus at an energy of 13.5 MeV for parity plus levels (we recall that the bin has a width of 10 keV) compared to the theoretical one (dashed curve) calculated with $\sigma = 5$. . Here, the total number of levels is less than 1000 (about 860) and we can conclude than the gaussian law is valid for a total number of levels of this order of magnitude.

Figure 3 :   Values of the spin cut-off parameter σ for every bins of energy, from 0 MeV to 28 MeV by step of 10 keV excluding bins with less than 100 levels, with parity $\pi = +1$ for the ^{50}Mn nucleus. For the big solid triangles, see text (§3-e).

Figure 4a :   Evolution of the parity distribution function $A(E)$ (see text for its definition) with the energy for the ^{56}Fe nucleus.

Figure 4b : The same as figure (4a) for the ^{50}Mn nucleus.

Figure 5 : Our simple modelisation of the parity distribution function $A(E)$ with the two key-energies E_1 and E_2.